Advanced Macroeconomics II

Assignment 1

(Submission Time: 3:00 pm, 11 June 2007)

1. For a neoclassical production function, show that the each factor of production earns its marginal product. Also show that if owners of capital save all their income and workers consume all their income, then the economy reaches the golden rule of capital accumulation. Explain the results.

2. Consider the Solow model with the following CES production function:

$$Y = \left[(a_F K_F^\eta + a_I K_I^\eta)^{\psi/\eta} + a_G k_G^\psi\right]^{1/\psi},$$

where Y is output; K_F is formal capital, which is subject to taxation; K_I is informal capital, which evades taxation; K_G is public capital, provided by the government and used freely by all producers; $a_i > 0$ $(i = F, I, G)$; $\eta < 1$ and $\psi < 1$. Installed formal and informal capital differ in their location and form of ownership and, therefore, in their productivity.

Output can used on a one-for-one basis for consumption or gross investment in the three types of capital. All three types of capital depreciate at the rate δ. Population is constant, and there is no technological progress.

Formal capital is subject to tax at the rate τ at the moment of its installation. Thus, the price of formal capital (in units of output) is $1 + \tau$. The price of a unit of informal capital is 1. Gross investment in public capital is the fixed fraction s_G of tax revenues. Any unused tax receipts are rebated to households in a lump-sum form. The sum of investment in the two forms of private capital is the fraction s of income net of taxes and transfers. Existing private capital can be converted on a one-for-one basis in either direction between formal and informal capital.

(a) Derive the ratio of informal to formal capital used by profit-maximizing producers.

(b) In the steady state, the three forms of capital grow at the same rate. What is the ratio of output to formal capital in the steady state?

(c) What is the steady-state growth rate of the economy?

(d) Use numerical simulations to show that, for reasonable parameter values, the graph of the growth rate against the tax rate, τ, initially increases, then reaches a peak, and finally decreases steadily. Explain this nonmonotonic relation between the growth rate and the tax rate.

3. Consider the Ramsey-Cass-Koopmans model discussed in class. Suppose that

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta} \quad \text{and} \quad Y = AK^\alpha L^{1-\alpha}$$

where $\theta > 0$, $0 < \alpha < 1$ and $A > 0$ (constant).

(a) Set up the household's optimization problem and use the current-value Hamiltonian to find the first-order conditions.

(b) Set up the firm's problem and find the first-order conditions.

(c) Find the steady-state values of k and c.

(d) Could the economy be dynamically inefficient? Explain.

(e) Construct a phase diagram in (k, λ) space to show that the steady state is a saddle point. Note that λ is the co-state variable associated with the household's optimization problem.